Annexure D

METHODS FOR ESTIMATION OF M_{max} WITH ILLUSTRATIVE EXAMPLES

This Annexure provides the details of the seven possible methods listed in Section 7.5 for estimating the maximum magnitude, M_{max}, for a seismic source zone (SSZ). An illustrative example of the estimation of M_{max} by each of the methods is also given.

1. Increasing the Largest Historical Earthquake by Suitable Magnitude Units

In this method, the M_{max} is obtained simply by increasing the largest observed earthquake magnitude, M_{max}^{obs}, in a source zone by 0.5 to 1.0 magnitude units with no definite guidelines available for deciding the amount of increment. If the M_{max}^{obs} in a source zone is say 6.5, M_{max} may have a value between 7.0 and 7.5, which is quite wide range. However, the increment should in reality be much less than 0.5 when the M_{max}^{obs} is already close to the maximum potential of the source zone. It is thus proposed to use a differential increment with an upper cap of 0.5 for the Himalaya and 1.0 for stable continental region of Peninsular India. In the Himalayan region, magnitude units of 0.5, 0.4, 0.3, 0.2, 0.1, and 0.0 may be added to M_{max}^{obs} in the ranges of ≤6.2, 6.3–6.8, 6.9– 7.3, 7.4–7.7, 7.8 – 8.1, and ≥ 8.2, respectively. Similarly, in the Peninsular India except the Kachchh region, magnitude units of 1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, and 0.0 may be added to M_{max}^{obs} in the ranges of ≤4.5, 4.6 – 4.8, 4.9 – 5.0, 5.1–5.2, 5.3– 5.4, 5.5– 5.6, 5.7– 5.8, 5.9–6.0, 6.1 – 6.2, 6.3 – 6.4, and ≥ 6.5, respectively. These increments are only indicative, and the actual value has to be decided judiciously from case to case by taking into account the factors like total duration of the available earthquake catalog, size of the source zone, regional maximum magnitude, and the tectonic environment of the source zone.

2. Extrapolation of G-R Relationship

Due to limited duration of the earthquake catalog, it is generally unlikely that the largest possible magnitude in a source zone is already included. Extrapolation of the frequency-magnitude relationship developed using the catalog data to a longer period may thus be used to get an estimate of the M_{max} (Bollinger et al., 1992; Wheeler, 2009). The extrapolation of the G-R relationship to a period of Y years greater than the duration of the catalog gives the maximum magnitude as

$$M_{max} = \frac{a + \log_{10} Y}{b} \tag{D.1}$$

For a source zone with $a = 3.9$ and $b = 0.9$ for a source zone with M_{max}^{obs} =6.5 and catalog duration of 102 years, the M_{max} for Y = 200, 500 and 1000 years are obtained as 6.9, 7.3 and 7.7, respectively. Though, it is difficult to decide accurately the period Y for the application of this method, a value of around two times the period of the catalog used to define the G-R relationship may be considered a good choice in practical engineering applications.

3. Statistics of the Ordered Largest Earthquake Magnitudes

Dargahi-Noubary (1999) has proposed a statistical method for estimation of $M_{\max}$ from knowledge of the magnitudes of a few largest earthquakes and an approximate knowledge of the total number of events in a source zone. If n is approximately the total number of events and $M_1, M_2, \cdots, M_n$ are their magnitudes arranged in decreasing order, then the value of $M_{\max}$ with a confidence level p can be defined by

$$M_{\max} = M_1 + \frac{(M_1 - M_2)}{p^{-\alpha} - 1}; \quad \text{with } \alpha = \frac{\ln k}{\ln[(M_3 - M_k)/(M_2 - M_3)]} \tag{D.2}$$

where $k = [\sqrt{n}]$ is the largest integer less than or equal to the number within the brackets. For example, in a typical source zone $n = 258$, which gives $k = 16$. Also, the available data in the source zone gives $M_1 = 6.5$, $M_2 = 6.0$, $M_3 = 5.9$ and $M_{16} = 5.4$. Using these values in Eq. (D.2) with $p = 0.63$ gives $M_{\max} = 6.9$, which is 0.4 magnitude units higher than the observed maximum magnitude. The value will be higher for higher confidence levels, but confidence level of 0.63 is considered appropriate for practical application of this method, which is the confidence level with which $M_{\max}$ can occur during its recurrence period under the Poisson assumption. It needs to be noted that that this method will not predict the $M_{\max}$ value higher than the $M_{\max}^{obs}$ if the second largest magnitude also has the same value. Thus, the difference between the two highest magnitudes may perhaps be used to rationalize the magnitude increment to be used in method-1.

4. Using Mixed Data Probability Distribution

Kijko and coworkers (1989, 1992, and 2016) have developed methods for getting the maximum likelihood estimates of the mean seismic activity rate λ, b-value in the G–R relation, and the $M_{\max}$ using mixed probability distributions for all the three parameters. Both, the incomplete and complete parts of the catalog with different magnitudes of completeness are used for the estimation of these parameters with their uncertainties accounted by Bayesian probability distributions. Kijko et al. (2016) have developed a MATLAB program to implement this method, which has been used to estimate the $M_{\max}$ using the available data on past earthquakes in a source zone. for the example source zone. The data for 258 earthquakes used for the source zone in the previous method gives $M_{\max}$ =6.68 ≈ 6.7.

5. Using Cumulative Strain Energy Plot

Earthquakes in a seismic source zone are generated by sudden release of the strain energy stored at a very slow rate over long period of time. The plot of the cumulative strain energy released in the form of earthquakes as a function of time is known as the Benioff plot, which can be used to estimate the probable maximum magnitude in a source zone as described in Makropoulos and Burton (1983). The energy E_i released by *i*th earthquake in the catalogue with magnitude M_i can be estimated from the relationship $\log E_i = 1.5M_i + 11.8$ due to

Gutenberg and Richter (1956). A typical plot of the cumulative energy $\sum E_i$ versus time in years for a source zone is given in Figure D.1.

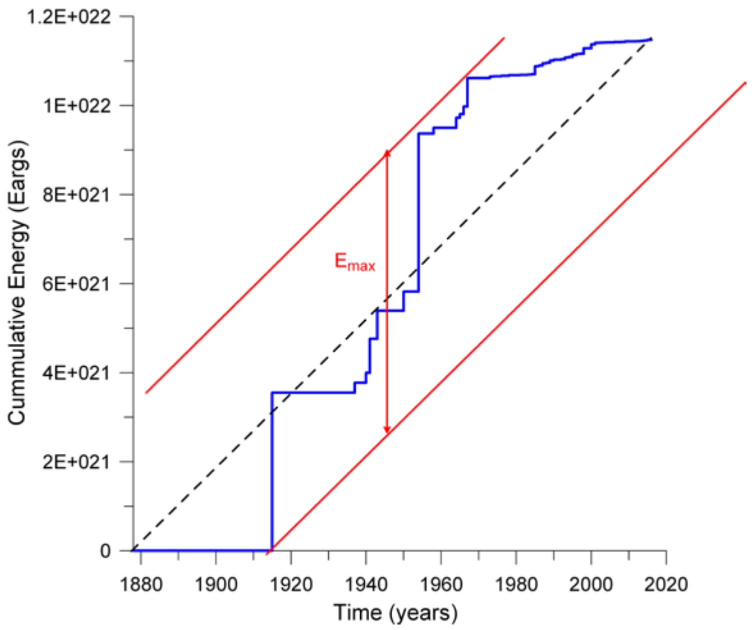

Figure D.1: A typical example of the cumulative energy released versus time.

The dashed line in Figure D.1 connects the starting and the end point, the slope of which represents the average rate of energy release with time. The vertical difference between the two red lines drawn parallel to the dashed line and enveloping the Benioff plot indicates the amount of maximum energy, $E_{\max}$, that may be released in a single earthquake in the source zone under consideration. This can be used to obtain the corresponding $M_{\max}$ from the relationship $\log E_{\max} = 1.5M_{\max} + 11.8$. In the plot of Figure D.1, $E_{\max} = 6.25 \times 10^{21}$ Ergs, which gives $M_{\max} = 6.67 \approx 6.7$.

6. Using Fault Rupture Length

Using a database of worldwide earthquakes, Wells and Coppersmith (1994) have developed empirical relations between earthquake magnitude, M_W, and the surface rupture length, L, and the subsurface rupture length, $\widehat{L}$, of the fault in km for different types of faults (SS=strike slip, RV=reverse, NR=normal, and UN=unspecified) as follows:

$$M_W = \begin{cases} 5.16 + 1.12\log L; & \text{SS} \\ 5.00 + 1.22\log L; & \text{RV} \\ 4.86 + 1.32\log L; & \text{NR} \\ 5.08 + 1.16\log L; & \text{UN} \end{cases} \text{ and } M_W = \begin{cases} 4.33 + 1.49\log \widehat{L}; & \text{SS} \\ 4.49 + 1.49\log \widehat{L}; & \text{RV} \\ 4.34 + 1.54\log \widehat{L}; & \text{NR} \\ 4.38 + 1.49\log \widehat{L}; & \text{UN} \end{cases} \qquad \text{(D.3)}$$

These relations can be used to estimate the maximum magnitude for a specific fault, provided the maximum surface or subsurface length of the fault that may rupture during a future earthquake is known. However, the surface and subsurface rupture lengths of expected future earthquakesin a source zone cannot be predicted with any reliability to implement this method. In practical applications, the maximum rupture length is generally taken as a small fraction of the total length of the fault trace on the surface (e.g., Mark, 1977; Slemmons, 1982; Kayabalia and Akinb, 2003). Correlating the rupture length estimated from the relationships in Eq. (D.3) for the observed past earthquakes in a region with the associated fault trace lengths, Anbazhagan et al. (2015) have proposed to define this fraction in a region-specific manner. However, the rupture length is seen to be highly uncertain to use this method confidently in practical applications.

7. Using strain rate data from GPS measurements

The strain rate data available from the local (Mukhopadhyay et al., 2020) or global sources (Kreemer et al., 2014) can be used to estimate the scalar moment rate, $\dot{M}_0$, for an area source (Kostrov, 1974), which in turn can be used to estimate the $M_{\max}$ from knowledge of the G-R parameters a and b using the following expression

$$\dot{M} = \left(\frac{c}{c-b}\right)\times 10^{a-bM_{\max}} \times 10^{cM_{\max}+d} \qquad \text{(D.4)}$$

This expression is equivalent to the original expression by Molnar (1979), developed on the basis of the G-R relationship and the moment magnitude relationship $\log_{10} M_0 = cM + d$ with $c = 1.5$ and $d = 16.1$ due to Hanks and Kanamori (1979).

Using the principal components ε_1 and ε_2 of the strain rates based on the GPS data over a rectangular grid of size δA, the scalar moment rate, $\delta\dot{M}_0$, which can be define as

$$\delta\dot{M}_0 = 2\mu H\delta A \max(|\varepsilon_1|,|\varepsilon_2|,|\varepsilon_1+\varepsilon_2|) \qquad \text{(D.5)}$$

where μ is the modulus of rigidity, H is the thickness of the seismogenic layer, δA is the area of the grid cell, and the function $\max$ is equal to the largest of its arguments. By summing the $\delta\dot{M}_0$ for all the grid cells in a source zone provides an estimate of the moment rate $\dot{M}_0$ for the source zone, which can be used in Eq. (D.4) to estimate the value of $M_{\max}$ for the source zone.

Using the strain rate data provided by Kreemer et al. (2014) for grid cells of size 0.25°× 0.20° in latitudes and longitudes for the entire globe and taking $\mu = 3.0\times 10^{11}$ dyne/cm^2 and the seismogenic thickness H as 15 km on the basis of the average focal depths of past earthquakes, the sum of the moment rates for all the grid cells in the example source zone used to obtain the example results by the other methods gives the moment rate of $\dot{M}_0 = 9.248\times 10^{24}$ dyne-cm/year. Using this value of along with the a and b values of 3.9 and 0.9, respectively, gives $M_{\max} = 7.6$ with a return period of about 870 years, which is 1.1 magnitude units higher than the maximum observed magnitude of 6.5. The $M_{\max}$ estimate may be much lower if it accounted that the entire source volume may not be seismogenic and that there may be significant creep type of slip in the strain rate.