STAGE-DISCHARGE RELATIONSHIPS, EXTRAPOLATION, EFFICIENCY, SHIFTING CONTROL, LIMITATIONS

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1.0 Abstract

The direct measurement of a hydrological time series has been one of the most complicated tasks due to wide range of data, uncertainties in the parameters influencing it and direct measurement of discharge in alluvial open channels is time consuming and costly (sometimes impractical during large floods). Most discharge records are developed from converting measured water stages to discharges by using a calibrated stage-discharge rating, which permits a fast and relatively inexpensive means to determine the discharge. In this lecture, an attempt has been made to elucidate through understanding of the stage-discharge relationship for a river for providing flow estimates given easy-to-obtain stage measurements, and flow level estimates given discharge predictions and other allied complexities involved.

1.1 Introduction

Discharge-rating curves showing the relationship between water-surface stage and the flow discharge. Despite being widely used, century-old tool, the underlying physics and scientific justification of the rating curve have not been adequately explored. This has resulted in widely recognized problems in developing and applying ratings, and a variety of empirical adjustments to address these problems. Rating curves are established by concurrent measurements of stage, h, and discharge, Q, (through velocity measurements, dilution methods, or other techniques) and the results are fitted to yield the rating curves. Discharge measurements need to cover the range of flows at the site and need to be continued over time to account for temporal changes in the rating. Stage-discharge ratings are generally treated as following a power curve of the form given by the following equation (Kennedy, 1984):

$$Q=c(a+h)^{\alpha} \qquad \ldots 1$$

where, α is an index exponent, and *a* and *c* are constants. The constant *a* is often referred to as the “offset” or as the “gage-height of zero flow.” For most stations the rating will be a compound curve consisting of different segments, each of which may follow the form of equation 1, but have unique values of *c, a,* and α. For most gauging stations the “offset” is a mathematical constant determined by successive approximations to obtain the best fit between measured discharges and stages. In reality, the discharge in a channel is generally a function of not only the stage**,** but also the **water-surface slope**, the **channel geometry, the unsteadiness of the flow**, and possibly other factors. In other words, the stage-discharge relationship is not unique but multi-valued, which is often seen by discontinuities or loops in rating curves. Several empirical methods have been developed to adjust discharges from ratings to account for these factors. Most of these methods are based on an estimate of the water-surface slope. These methods empirically establish a reference single-curve stage-discharge relationship for a specified “reference” or “normal” fall or steady-flow condition. “Normal” has been used by many investigators to refer to a typical condition, rather than hydraulic normal (steady uniform) flow, as defined in open-channel hydraulics. The fall is the difference in water elevation between two defined points along the channel. The “reference” fall may either be arbitrary or a fall selected based on observation of the discharge-measurement record. An auxiliary relation gives an empirical fit between the ratio of discharges to some other factor (e.g., the ratio of measured to reference falls or the rate of change of stage divided by the reference slope). This discharge ratio is used, along with the discharge from the reference stage-discharge rating, to determine the discharge for conditions other than the reference conditions. These methods require many measurements to fit the rating and auxiliary relations, because the measurements need to be suitably distributed throughout the range of water levels, slopes, and rates of rise and fall of the hydrograph. These methods implicitly assume that the steady or “normal” discharge reflects uniform flow conditions and that the other factors affecting the flow are represented by the slope or rate of change of stage and the velocity of the flood wave.

Almost universally the routine measurement of the state of a river is that of the stage, the surface elevation at a gauging station, usually specified relative to an arbitrary local datum. While surface elevation is an important quantity in determining the danger of flooding, another important quantity is the actual flow rate past the gauging station. Accurate knowledge of this instantaneous discharge - and its time integral, the total volume of flow - is crucial to many hydrologic investigations and to practical operations of a river and its chief environmental and commercial resource, its water. Examples include decisions on the allocation of water resources, the design of reservoirs and their associated spillways, the calibration of models, and the interaction with other computational components of a network.

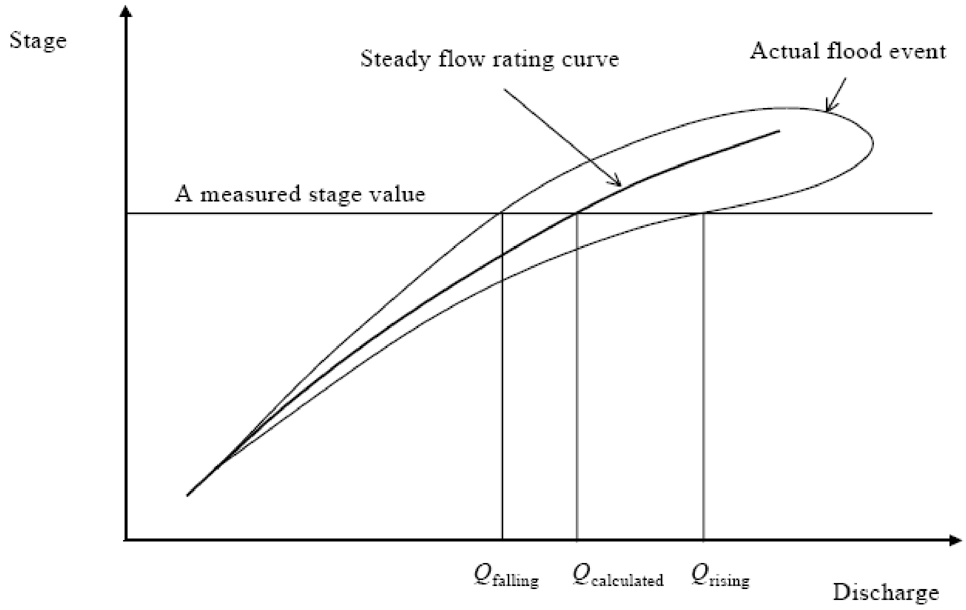

Fig- Stage-discharge diagram showing the steady-flow rating curve and an exaggerated looped trajectory of a particular flood event

1.2 Theoretical background

A stage-discharge relation provides an estimate of the flow past a cross section based on a point measurement- namely the stage at the location of the stage sensor. This point estimate combines the equation for the discharge through a cross section with an equation for the mean velocity.

Thus, the theoretical basis of the rating curve starts from equation 2:

$$Q = AV = AK\sqrt{RS_f} = \left(A\sqrt{R}\right)K\sqrt{S_f}$$

… 2

in which *Sf* is the friction slope, *A* is the flow cross sectional area, *V* is the mean velocity in the cross section, *R* is the hydraulic radius, and the coefficient *K* depends on the flow formula used,

$$K=C \text{ (Chezy)}, \quad K=\sqrt{8g/f} \text{ (Darcy-Weisbach)}, \quad K=\frac{K_n}{gn}R^{1/6}$$

.. 3

f gn (Manning), C, f, and *n* are the Chezy, Darcy-Weisbach, and Manning resistance coefficients, respectively; *g* is the acceleration of gravity; and *Kn* is a units-correction constant for Manning's n (Yen, 1992). The product *R A* is a function of the water stage h. This product has historically been assumed constant for a given stage for fixed-bed channels. The friction slope can be determined from the one-dimensional momentum equation for unsteady, gradually varied flow (eqn. 3):

$$S_f = S_o - \left[\frac{1}{g}\frac{\partial V}{\partial t} \quad + \quad \frac{V}{g}\frac{\partial V}{\partial x} \quad + \quad \frac{\partial h}{\partial x} \right]$$

a **b** **c** **d** **e**

˜4

This equation, combined with the continuity equation describing conservation of mass (Eqn. 5), form the Saint-Venant, or complete dynamic wave equations for unsteady flow (Yen, 1973, 1979):

$$\frac{\partial h}{\partial t} + D\frac{\partial V}{\partial x} + V\frac{\partial h}{\partial x} = 0$$
$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = T\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

…5

in which, S_o is the channel bottom slope, *S0=-.z/.*x; *t* is the time; *x* is the longitudinal coordinate along the channel in the horizontal direction; *h* is the depth, measured in the *z* (vertical) direction; *D = A/T* is the hydraulic depth; *T* is the free-surface widt*h;* and all other terms have been previously defined. The letters below equation 3 defines the terms in these equations for subsequent discussion. Term **a** is the friction slope and reflects the resistance to flow. Term **b** is the bed slope and reflects the body force from gravity. Term **c** is the local acceleration and reflects unsteady flow. Term **d** is the convective acceleration and reflects both spatial variation of the flow (.Q/.x) and longitudinal change in the cross-section area (.A/.x). Term **e** is the pressure term and reflects the change in depth in the longitudinal direction. Terms **b** and **e** combine to give the water-surface slope. From eq^{n}s. (3) and (4) it is clear that the discharge is a function of not only the stage but also the water-surface slope, change of area along the channel, and unsteadiness.

1.3 Stage-Discharge Controls

Stage discharge relation is controlled by a station control, a section or reach of channel downstream from the gauge. A section control is usually effective only at low discharges. At medium and high discharges, section controls are completely submerged, and the relationship between stage and discharge is governed by channel control. Channel control is the set of all physical features of the channel that dictate the river stage at a given point for a given flow rate. The features include size, slope, roughness, alignment, constrictions and expansions, and the channel shape. The channel reach that functions as a control may lengthen as the discharge increases, introducing new features in the stage discharge relationship.

The development of the rating curve when there is more than one control effective and when data are limited, requires judgment in both interpolation and extrapolation of the data. This situation is partially aggravated when the controls are not permanent; the various discharge measurements are then representative of changes in the positioning of the segments of the rating curve.

1.4 Development of Rating Curves

Discharge is plotted as the abscissa; gage height or stage, the ordinate. The discharge measurements are numbered consecutively in chronological order to facilitate the identification of time trends.

Hand drawn curves are typically used to fit the stage and discharge measurements to produce a rating curve. Considerable judgment is normally exercised to decide on the best curve. For example, knowledge of the river is applied and consideration is given to such factors as the quality and magnitude of each measurement. In the simplest case, when a single control exists, these curves would be practically straight lines on log scales. The case of compound controls will be discussed later in the lesson.

After the hand drawn curve is established, representative points that lie exactly on the curve are extracted. Figure 3 illustrates how these points lie on the curve in contrast to the actual stage- discharge measurements. Once the defining points have been selected, one or more logarithmic equations are used to mathematically fit the points. It is from these logarithmic equations that the actual rating table values are derived, usually at increments of 0.3 cm of stage as shown in Figure 2.

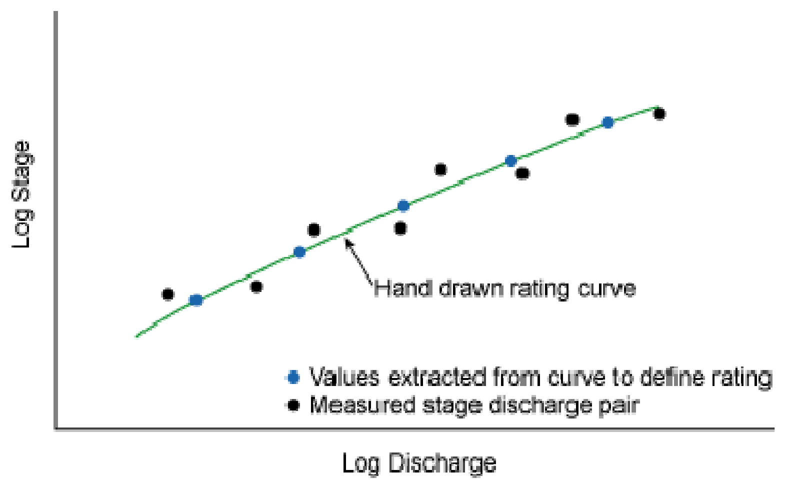

Figure 1. Hand drawn rating curve with 5 values extracted from the curve to define the rating.

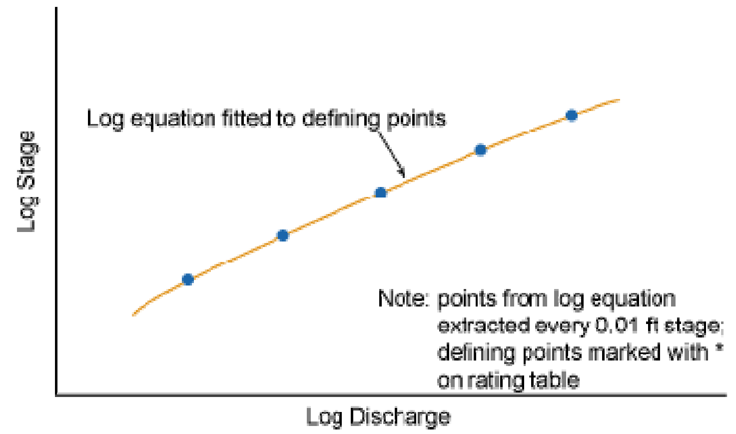

Figure 2. Rating curve with log equation through defining points.

1.5 Interpreting the Rating Curve

Paired stage and discharge data are commonly plotted on logarithmic paper (log scale for both the ordinate and abscissa) since this scaling tends to produce a nearly linear (or at least piece-wise linear) fit to the observed data. Assuming the rating "curve" is nearly a straight line in situations with compound controls, changes in the slope of log rating "curve" identify the range in stage over which the individual control is effective. The "linearized" rating line also makes extrapolation or interpolation comparatively easy compared to using the data on an arithmetic or rectangular scale. The benefit of using rectangular-coordinate paper for rating analysis is that trends and changes in the low flow portion of the curve are more apparent and that zero flow conditions can be identified. Zero flow cannot be described in the log scale. For flood forecasting, however, logarithmic plotting is preferable because of its ability to identify control changes.

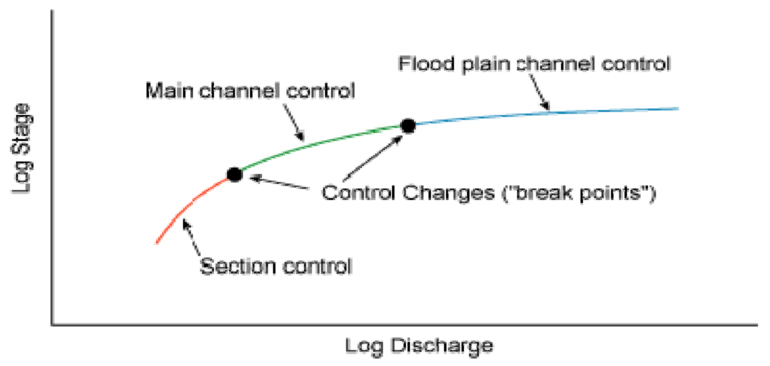

Figure 3. Rating curve for a stream with section, channel and flood plain controls.

Figure 3 shows a rating curve with three distinct sections within which the segments can be described as nearly linear. The point at which the slope of the curve changes coincides with the stage at which one control becomes submerged and the next control becomes effective. This transition point should be considered when observed data is being monitored during significant river rises. For example, consider the case where a river stage is increasing at a dangerous rate, and it is occurring at a stage that is near the top of the portion of the rating under the main channel section control. It can be concluded from the rating curve that the rate of rise will likely moderate even if the rain and flow rate remain constant. From Figure 3, it can be seen that significant increases in flow are required for only moderate increases in stage once the stage reaches a level that is under the influence of the flood plain channel control portion of the rating. Figure 4 relates this concept to the channel characteristics. Once the flow leaves the main channel, the horizontal component of the cross sectional area increases dramatically, compared to the main channel. Correspondingly, the vertical component (or stage) of the cross sectional area does not increase significantly for increases in flow. For these reasons, ratings plotted logarithmically provide useful insight into the behavior of a river reach at moderate to high flow.

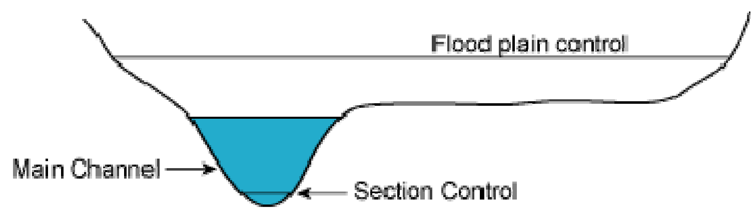

Figure 4. Channel cross section showing section, channel, and flood plain control stage heights.

1.6 Rating Curves for Artificial Section Controls

The rationale for addressing rating curves for artificial controls is that these controls are the basis of stage discharge relationships for natural controls. Thin plate or sharp-crested weirs, broad-crested weirs, and flumes are the most common artificial controls.

The crest of a thin plate weir is susceptible to damage from floating debris, so they are generally only used in small, clear-flowing streams. Broad crested weirs are used in larger streams. Both thin plate and broad-crested weirs provide high accuracy although regular maintenance is required. In contrast, flumes are preferable for small streams with significant sediment and debris loads; they are also where the head loss associated with thin plate weirs is unacceptable.

1.7 Broad Crested Weirs

The most common artificial control built in natural channels is the broad crested or flat crested weir. These types of weirs have the strength and durability to withstand debris damage. They are often built with a gently sloping upstream apron (1V :5H) to minimize and impedance to flow and sediment carried over the weir. The weirs are usually built low so that they can act low water controls; they are submerged at intermediate and high stages.

1.8 Flat Crested Rectangular Weir

The most basic type of broad crested weir is the rectangular in cross section weir . The discharge equation can be expressed as

$Q = CL(h + h_v)^{1.5}$ ..6

where Q is the discharge, C is a discharge coefficient, b is the width, h is the head, and h_r is the head attributable to the approach velocity. The discharge coefficient, C, and h_v both increase with stage as shown in Figure 5.

The rating curve when plotted on log paper with again be a straight line, except for extremely low stages. The equation can be represented as

$Q = p(G - e)^N$ ..7

Where, the exponent is the slope of the curve. The exponent will exceed 1.5 since the velocity head and the weir coefficient increase with stage.

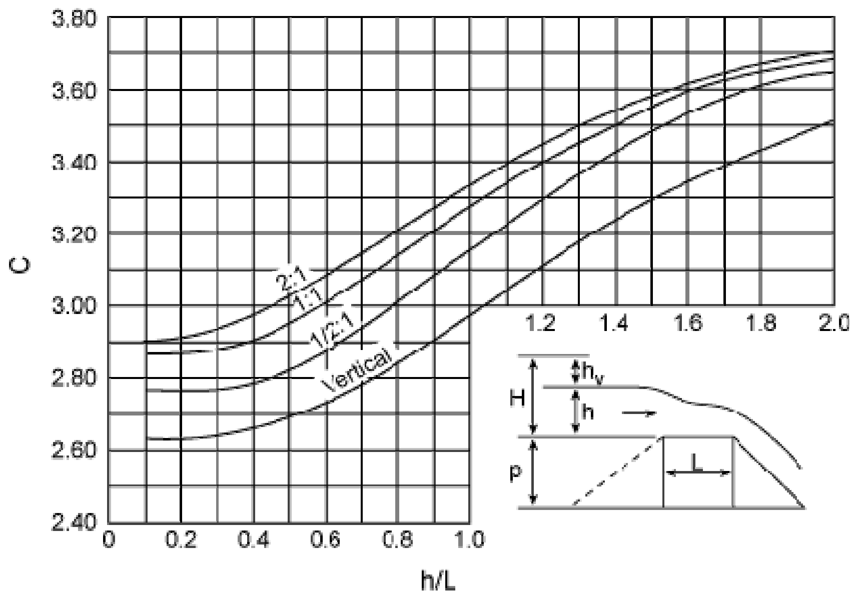

Figure 5. Rectangular flat crested weir.

1.9 Trenton Type Control

If the crest is horizontal, the stage discharge equation is

$Q = 3.5bh^{1.65}$ ...8

The constants will vary with the height of the weir above the streambed; they are greater than those for a flat crested weir because the cross section is more efficient in water flow.

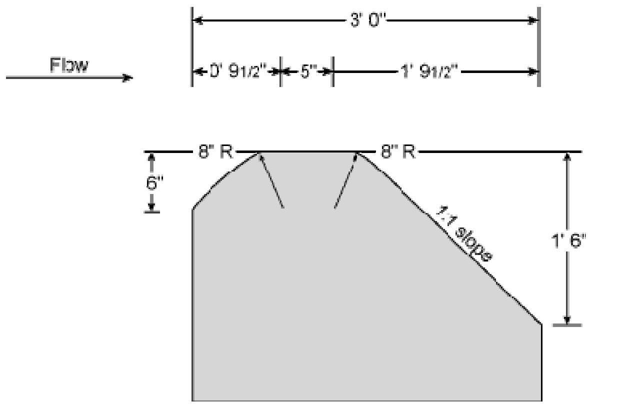

Figure 6. Trenton type control in concrete broad crested weir.

1.10 Columbus Type Control

Another common type of control in the United States is the Columbus type control. The control is also concrete but with a parabolic notch that gives more accurate measurement of a wide range of flows. The Q equation is

$$Q = 38.5(h - 0.2)^{3.3} \qquad ..9$$

The equation assumes the head exceeds 21.2 cm, the elevation of the top of the notch. This implies the elevation of effective zero flow is 6 cm.

1.11 Submergence of Broad Crested Weirs

Similar to thin plate weirs for a given static head, the discharge in broad crested weirs decreases as the submergence ratio increases. The threshold value of the submergence ratio, where the discharge is first impacted, ranges from 0.65 to 0.85, depending on the cross section shape of the weir crest.

1.12 Flumes

Flumes are commonly used in contractions in channel width where free fall or an increase in bed slope produces critical or supercritical flow in the flume. The stage measured at some cross section and discharge is only dependent upon the characteristics of the flume. Historically, one of the most common types of flumes was the Parshall flume, but it has generally been replaced with a variety of long-throated flumes. The most common supercritical flow flume is the trapezoidal flume. Flumes are commonly used in irrigation, flood, and water supply canals, but are rarely used for measuring discharge at gauging stations used for flood forecasting.

1.13 Natural Section Controls

If the control is a rock outcrop, riffle, or gravel bar, the stage discharge relationship is simulated as a broad crested artificial control.

The stage discharge relationship is again expressed by

$$Q = p(G - e)^N \qquad ..10$$

However, *N*, will be greater than 1.5, since the approach velocity increases with stage. If the crest has a roughly parabolic profile, the exponent will be even larger because of the increase in width of the stream with stage. The value of *N* will usually exceed 2.0. For irregular controls, the gage height of effective zero flow (*e*) for all but the lowest stages, will be greater than that for the lowest point in the notch.

1.14 Stable Channels

The stage discharge relationship for these channels is the Manning's discharge equation (in English units),

$$Q = (1.486AR^{2/3}S^{1/2})/n \qquad . ..11$$

Where, again *Q* is the flow rate, *A* is the cross sectional area of flow, *n* is the roughness coefficient, *R* is the wetted perimeter, and *S* is the streambed slope. For any stage, all quantities on the right hand side of the equation are known except the roughness coefficient. A value of *n* can be determined from a single discharge measurement, or an average value of *n* can be computed from a pair of measurements. A preliminary rating curve can be computed for the entire range of stage. If subsequent discharge measurements depart from the rating curve, it is likely that the assumption of the flow at uniform depth was invalid. In this situation, the energy slope, *S*, is not parallel to the bed slope, but varies with stage and consequently the value of *n*, computed on the basis of the bed slope, is in error.

In a natural channel of irregular shape, the stage discharge relationship can again be developed via Manning's equation. However, the following assumptions are necessary:

1. at higher stream stages, *n* is constant and the energy slope is constant
2. the cross sectional area is approximately equal to the depth (*D*) multiplied by the width (*W*)

Replacing $(1.486S^{1/2})/n$ with a constant C_1, Manning's equation becomes,

$$Q = C_1DWR^{2/3} \qquad ...12$$

Assuming the hydraulic radius, *R*, is equal to *D*, and *W* is constant, the equation simplifies to

$$Q = C(G - e)^{1.67} \qquad ...13$$

Unless the stream is exceptionally wide, *R* is much less than *D*. This has the effect of decreasing the exponent in the Manning equation; this reduction can be offset partially by increasing *S* or *W* with discharge. Changes in the roughness coefficient will also impact the exponent. The discharge equation can be expressed as

$Q = C(G - e)^N$ ...14

The exponent, *N*, varies between 1.3 and 1.8.

1.15 Compound (Section and Channel) Controls

Compound controls usually control the stage discharge relationship. Section control is effective for the lower stages and channel control, the higher stages. Section control is almost always characterized by an increasing second derivative of *Q* with respect to water depth *H*., while for channel control, the second derivative of *Q* with respect to water depth *H* is decreasing. This condition results in a linear (log plots) rating curves for section control almost always having a slope greater than 2.0 while the slope of the rating curve for channel controls have slope less than 2.0. A non rigorous proof of this relationship between the second derivative of discharge with water depth and rating curve slope can be developed by considering the general discharge equation, $Q = CH^N$ where *N* is again the slope of the line. The first derivative of the equation is

$dQ/dH = CNH^{N-1}$ ...15

The second derivative can be expressed as,

$d^2Q/dH^2 = CN(N-1)H^{N-2}$ ...16

The second derivative increases with stage provided *N* exceeds 2.0; it decreases with stage when *N* is less than 2.0..

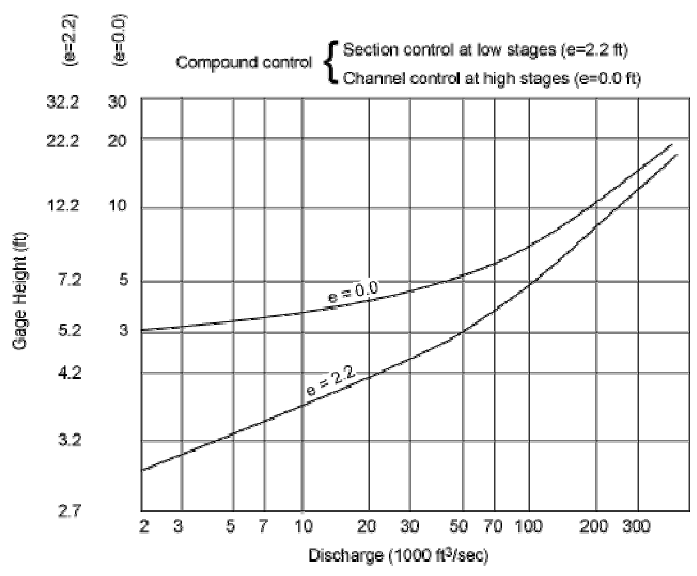

Figure 7. Compound control rating curve

1.16 Extrapolation

In practice, ratings curves are often extended or extrapolated beyond the range of discharge measurements. The extrapolation of the rating data can lead to considerable uncertainty in the prediction of discharge.

1.17 Low Flow Extrapolation

Low flow extrapolation is performed using rectangular coordinate graph paper since the coordinates of zero flow can not be plotted on the log graph paper. If the existing trend in the rating curve is extended to the zero discharge point, the curve will rarely pass through the zero stage point. Forcing the rating curve through the zero/zero stage-discharge point usually requires a different shaped curve then in the observed portion of the rating curve. Under these conditions, an adequate understanding of the relationship between low stage and discharge can only be achieved with additional low flow discharge measurements.

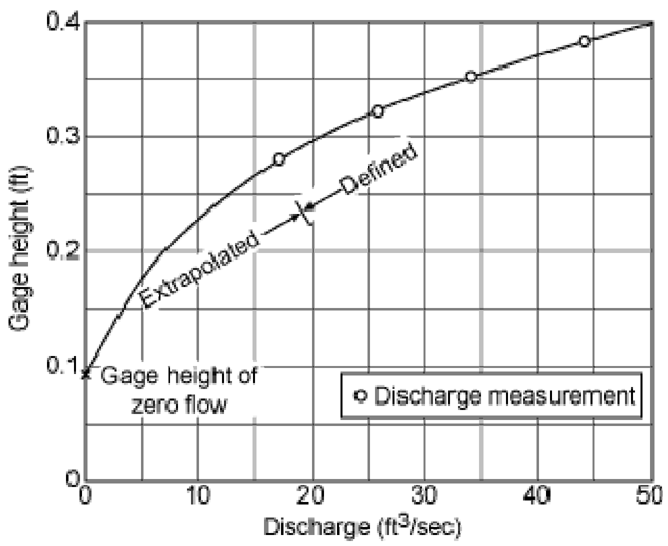

Figure 8. Rating curve extrapolation.

1.18 High Flow Extrapolation

The ramifications of high flow extrapolation are potentially more severe than low flow extrapolation. Errors in high flow extrapolation can underestimate flood peaks with the consequent loss of human life and property. Five methodologies are used for high flow extrapolation:

1. indirect determination of peak discharge
2. conveyance-slope method
3. area comparison of peak runoff rates
4. step backwater method
5. flood routing

1.19 Shifts in the Discharge Rating

During periods of shifting control, frequent discharge measurements are essential to identify the stage discharge relation and the magnitude of the shifts during the period. Even with limited data, the rating curve can be estimated if available measurements are supplemented with the behavior of the shifting control. It is this behavior which we will now address.

1.20 Detection of Shifts

Discharge measurements that lie within plus or minus 5 percent of the discharge value of the rating curve are considered to be a verification of the rating curve. If several consecutive measurements satisfy the 5 percent criterion, but they all lie on the same side of a segment of a rating curve, they are considered to define a period of shifting control. If the plus or minus 5 percent criterion is too stringent, departures are acceptable if the indicated shift does not exceed 0.06 cm.

It is also possible to quantify the shift in control using the standard deviation of the stage discharge rating curve. If the rating curve is divided into *j* segments, the standard deviation of the measurements about the rating curve in the j^{th} segment, s_j can be computed as

$$s_j = [\sum d_i^2/(N-1)]^{1/2} \qquad \text{...17}$$

where d_i is the departure of the i^{th} discharge measurement from the rating curve (percent) and *N* is the number of measurement used to define the j^{th} segment of the rating curve.

1.21 Procedures for Shift Adjusting Ratings

Once a sequence of paired stage/discharge measurements indicate that a change in the relationship between stage and discharge over some range of stage values, a new rating curve is drawn. The new curve incorporates the shift in relevant portion of the rating and the original unshifted portion of the official rating curve. Three points on the hand drawn shift curve are typically chosen to characterize the shift. The points are reported as an input, or discharge for which the shift occurs, and the shift, or difference between the official rating and the shifted curve. The values of the input and shift are used to communicate the nature of the shift.

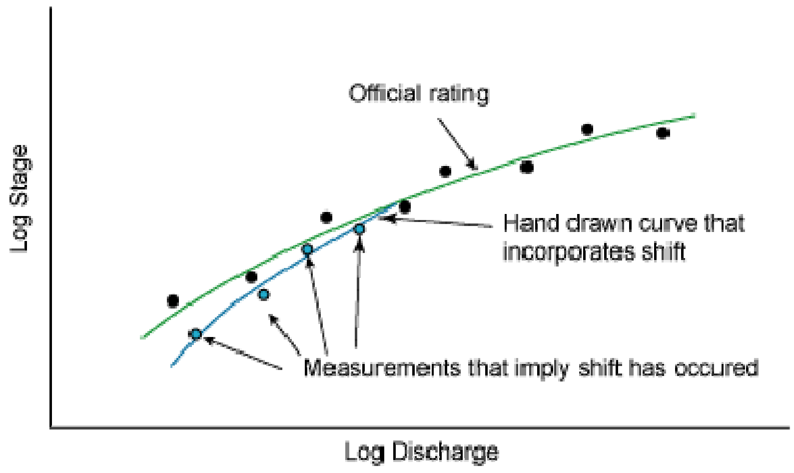

Figure 9. Rating curve with a hand drawn shift.

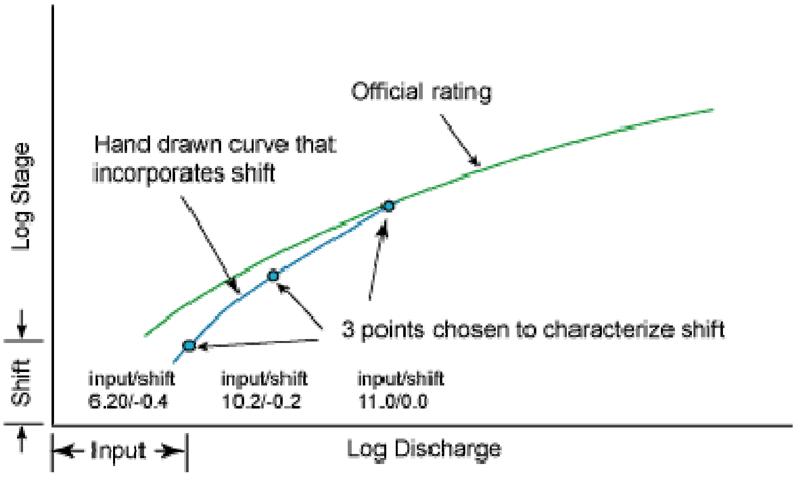

Figure 10. Rating curve with three defining points selected from the hand drawn shift .

Once the three defining points are selected, a new logarithmic function is determined that passes through these three points. For stages beyond the influence of the shift, the existing, unshifted, rating values are used.

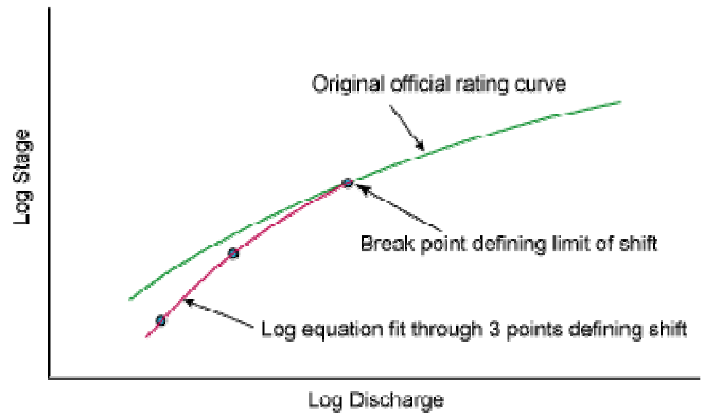

Figure 11. Rating curve with a shift defined by a logarithmic function.

Adequately defining the entire range of a high flow shift is frequently difficult due to the infrequency of high flows and the difficulty of taking high flow discharge measurements. Figure 12 illustrates a shift occurring for high flow. In this case, measurements are not available to define the high end of the shift. The shift is characterized by prorating the values between the three defining points and extracting rating values at 0.032 cm increments. For stages above the shift, the original rating values are used after they are adjusted by the amount of the shift associated with the highest stage. The original rating curve and the shift adjusted curve would therefore be parallel in rectangular coordinates above the last shift point. If Figure 12, these curves are shown as nearly parallel for illustrative purposes but in log coordinates, they would appear to converge. The same approach is applied to develop rating values below the shift point associated with the lowest stage if lower flow measurements are not available to define the lower portion of the shift.

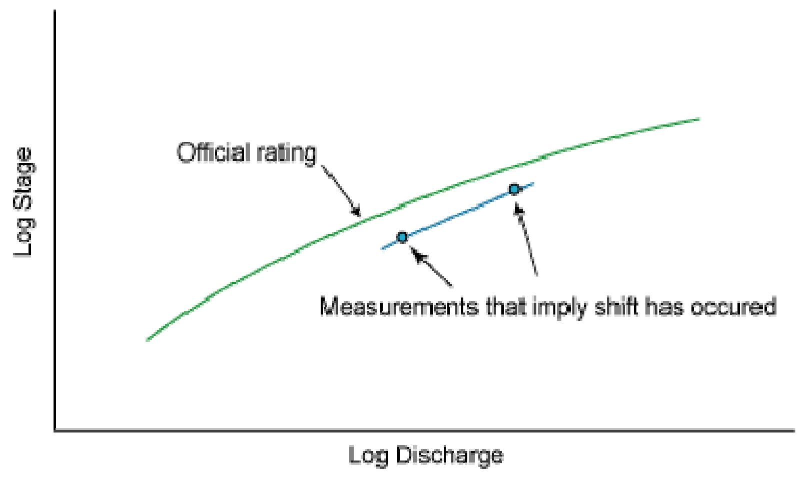

Figure 12. Rating curve with a high flow shift.

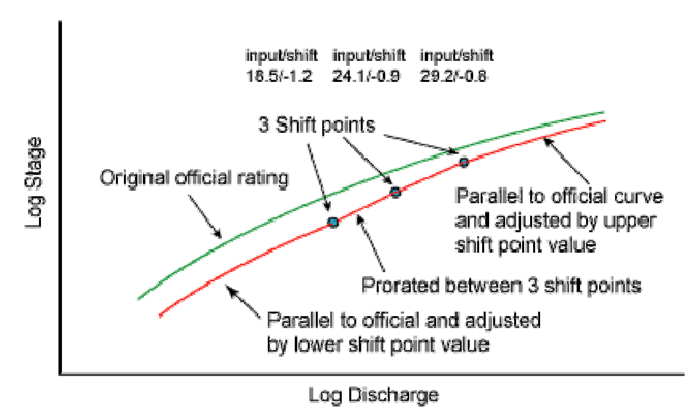

Figure 13. High flow shifted rating curve.

Stage fall discharge ratings are of two general types:

1. Rating fall constant—these relationships are developed for uniform channels; the water surface profile between gauging stations does not exhibit appreciable curvature.
2. Rating fall, stage dependent—these relationships are developed if
3. there is significant curvature of the water surface,

a. the reach is non uniform,

b. a submerged section control exists in the reach but the control does not become completed submerged even at high discharges, or

c. a combination of these factors.

There are two general principles that "govern" backwater effect. The first principle is that for a given stage at the variable control element, backwater decreases at the base gage as the discharge increases. Secondly, backwater decreases at the base gage as stage decreases at the variable control element; this presumes a constant discharge.

1.22 Rating Fall Constant

The water surface profile is parallel to the stream bed in uniform stream channels. The rating fall, F_r in uniform flow (no variable backwater) is the same at any stage. The stage discharge relationship can then be described by the Chezy equation,

$$Q=CA(RS)^{0.5} \quad \ldots 18$$

Where, C is the Chezy coefficient that represents the frictional losses occurring as water flows over the bottom and sides of the stream channel. It is related to the Darcy friction factor, f, that is used in pipe flow, or

$C=(8g/f)^{0.5}$ …19

Where, g is the gravitational coefficient. R is the hydraulic radius, e.g. the cross sectional area divided by the wetted perimeter, P, and S is the energy slope. In uniform flow, S is the slope of the channel bed. The energy slope can also be represented as $S=F_r/L$, where r denotes the base rating condition.

If a downstream tributary imposes variable backwater on the stream reach, the measured fall, F_m, and discharge, Q_m is less at a given stage than the given by the uniform discharge equation. Assuming the measured slope or fall represents the slope at the base gage, the stage discharge curves define a family of curves that are constant except for differing values of fall. The relationship can be summarized as

$Q/Q_r=(F/F_r)^{0.5}$ …20

The variable backwater discharge can then be expressed from Equation 21,

$Q=Q_r(F/F_r)^{0.5}$ …21

In situations where the base rating is controlled by a downstream dam, a constant rating fall can also occur. The water surface profile will again be approximately parallel to the channel bed at all discharges provide the curvature in the backwater profile is not significant. Typically F_r is assumed to be equal to 30.28 cm. In this special case, Eqn. 21 reduces to

$Q=Q_r F^{0.5}$, known as the ***unit fall method***.

Although, a constant rating fall is not usually encountered in natural streams, if discharge measurements cover the entire range of stream flow, there is no need to use more complicated equations. The measurements however have to conform to a constant rating fall. Theoretically this occurs when the curvature of the profile and the velocity head changes are truly negligible.

1.23 Discharge Determination

Three graphical relationships are necessary to determine the discharge. They include:

1. the relationship between stage and rating fall, F_r
2. stage versus rating discharge, Q_r
3. Q_m/Q_r versus F_m/F_r

The computation of discharge, Q_m, for a given stage and a given fall, F_m consists of the following steps:

1. determine the fall rating, F_r, for the known stage

2. compute the ratio F_m/F_r

3. using tabular representation of the data, determine the ratio, Q_m/Q_r

4. from the stage discharge information, determine the rating discharge, Q_r, for the known stage

5. determine $Q_m=(Q_m/Q_r)Q_r$

1.24 Rating Curves and Computer Models

Mathematical simulation models of open channel flow can be used for the prediction of stream discharge for a range of hydrologic events. In conjunction with observational data, the parameters of the model could be estimated using conventional parameter identification techniques.

A simplified example of the approach can be illustrated by considering Manning's equation. It is assumed that the acceleration head is negligible. Discharge measurements are made for the sole purpose of determining Manning's roughness coefficient, *n*. The estimate of *n* is not the true value of the parameter; it incorporates all the possible error sources in the flow equation, including the energy slope variation in the reach. Typically, the computed value of *n* would vary with stage.

Manning's equation can be expressed as

$$Q=KS^{0.5} \qquad \ldots 22$$

Where, *Q* is the discharge, *S* is the energy gradient, and *K* is the conveyance,

$$K=\delta/nAR^{2/3} \qquad \ldots 23$$

Where, *A* is the cross sectional area and *R* is the hydraulic radius. Equation 22 can be expanded to a reach as

$$Q = K_2 \left[\frac{F}{\frac{K_2}{K_1}L + \frac{K_2^2}{2gA_2^2}\left[-\alpha_1\left(\frac{A_2}{A_1}\right)^2(1-k) + \alpha_2(1-k)\right]} \right]^{1/2} \qquad \ldots 24$$

Where, the subscripts 1 and 2 denote the upstream and downstream cross sections respectively. *F* is the fall in the reach, *L* is the reach's length, *g* is the acceleration constant, *α* is the velocity head coefficient; its value is a function of the reach's velocity distribution. *k* is an energy loss coefficient; it is zero for contracting reaches and 0.5 for expanding reaches.

Data for the reach would define the relation between the stage and *K*, *A*, and *α*. Given the stage data, Equation above is solved to compute *Q*.

Rating curves in tidal reaches. These are the methods used for the purpose.

1. Method of cubatures
2. Rating fall method
3. Tide correction method
4. Coaxial graphical correlation method

1.25 Problems associated a Rating Curve:

• The assumption of a unique relationship between stage and discharge is, in general, not justified.

• Discharge is rarely measured during a flood, and the quality of data at the high flow end of the curve might be quite poor.

• It is usually some sort of line of best fit through a sample made up of a number of points – sometimes extrapolated for higher stages.

• It has to describe a range of variation from no flow through small but typical flows to very large extreme flood events.

• There are a number of factors which might cause the rating curve not to give the actual discharge, some of which will vary with time. Factors affecting the rating curve include:

– The channel changing as a result of modification due to dredging, bridge construction, or vegetation growth.

– Sediment transport - where the bed is in motion, which can have an effect over a single flood event, because the effective bed roughness can change during the event. As a flood increases, any bed forms present will tend to become larger and increase the effective roughness, so that friction is greater after the flood peak than before, so that the corresponding discharge for a given stage height will be less after the peak. This will contribute to a flood event showing a looped curve on a stage-discharge diagram**–** Backwater effects - changes in the conditions downstream such as the construction of a dam or flooding in the next waterway.

– Unsteadiness - in general the discharge will change rapidly during a flood, and the slope of the water surface will be different from that for a constant stage, depending on whether the discharge is increasing or decreasing, also contributing to a flood event appearing as a loop on a stage-discharge diagram.

– Variable channel storage - where the stream overflows onto flood plains during high discharges, giving rise to different slopes and to unsteadiness effects.

– Vegetation - changing the roughness and hence changing the stage-discharge relation.

– Ice - which we can ignore – this is India, after all.

Considerable uncertainty may be associated with the rating curve for a particular flow regime or river channel condition. For example, many channel cross sections change considerable throughout the year, especially after a significant runoff event. Any change in the channel cross section can have a significant effect on the relationship between stage and discharge. The rating curves before and after the flood shown in Figure 14 illustrates how significant the change in the rating curve can be after a single flood event.

The discharge rating curve transforms the stage data to a continuous record of stream discharge. The rating curve is also used to transform forecasted flow hydrographs into stage hydrographs. The discharge rating curve may be simple or complex depending on the river reach and flow regime.

Discharge models, also known as rating curves, stage ratings or stage-discharge relations are typically developed empirically from periodic measurements of stage and discharge. Discharge is computed current meter data. These data are plotted versus the concurrent stage to define the rating curve for the stream.

For new gauging stations, many discharge measurements are needed to develop the stage discharge relation throughout the entire range of stream flow data.

Generally periodic measurements are needed to validate the underlying stage-discharge relationship and to track changes or shifts in the rating curve. The USGS recommends a minimum of 10 discharge measurements per year, unless it has been demonstrated that the stage discharge relation is invariant in time. Of extreme importance is the capability of the stage-discharge relation to be applicable for flood or extreme flow conditions and for periods when the rating shifts as a result of ice formation.

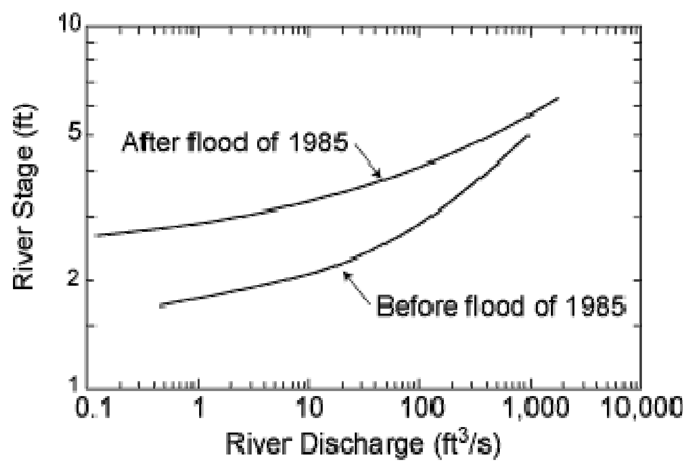

Figure 14. Stage-discharge curve before and after the flood

Discharge measurements are usually lacking in the definition of the upper end of the rating curve. As a result, the extrapolation of the lower parts of the rating curve is used to "approximate" the higher stages of the river. The extrapolation of these data is subject to serious error that can have significant implications for flood planning and the attendant loss in human life and property. The extrapolation issues can be circumvented, if indirect methods of determining unmeasured peak discharge are used. Without such data, some of the uncertainty can be reduced by estimating discharge associated with these stage values.

2.0 TESTING OF STAGE-DISCHARGE CURVES

The stage-discharge curves are to be tested for absence from *bias*, for *goodness of fit*, and for *shifts in control*. These tests are to be applied to the portions of the curves between the shifts in control, each individual portion being tested separately. As already discussed in previous module, it may not always be possible to fit a single mathematical equation for the entire range of stages and in many natural erodible channels (as is the case for most of the rivers in India), separate controls come into operation and cause composite curves with inflexions and discontinuities and in such a case a curve fit by eye may be best fit. The following tests are to be performed on the finalized stage-discharge curve.

2.1 Test -1

In a bias free curve drawn through 'N' observations, an equal number of observations are expected to be on either side of the curve. The actual number of points lying on either side should not deviate from N/2 by more than that can be explained by chance fluctuations in a binomially distributed variate with 1/2 as the probability of success. This is a very simple test and can be performed by counting the observed points falling on either side of the curve. If QO is the observed value and QE is the estimated value, then (QO - QE) should have an equal chance of being positive or negative. In other words, the probability of(QO - QE) being positive is 1/2. Hence assuming the successive signs to be independent of each other, the sequence of the differences may be considered as distributed according to the binomial law (p+q)N, where N is number of observations, and p and q are the probabilities of occurrence of positive and negative values which are one-half each. For N greater than 30, a value of 't' (a statistical parameter) lower than 1.96 indicates that the difference is not statistically significant at the 5% level.

Test 1 **- Test for number of positive and negative deviations**

S.No.	*Particulars*	*Symbol*	*Rising*	*Falling*
1.	Number of positive signs i.e points lying to the right side	n1		
2.	Total number of obs	N		
3.	Probability of a sign being	p	1/2	1/2
4.	Probability of a sign being	q	1/2	1/2
5.	Expected number of +ve	N.p		
7.	| n1 - N.P | - 0.5 * -------------------- N.p.q	t		

If the values of 't' for both the curves fit for rising and falling stages are less than

1.96, then these curves are free from bias as judged by this test.

2.2 Test - 2

This test will not only ensure a balanced fit with regard to the deviations over different stages, but will also help in detecting changes in control at different stages . The discharge measurements shall be arranged in the ascending order of stage for this test. For a good graduation, a sign change in deviation is as likely as a non-change of sign giving rise to a binomial distribution with parameters (N-1) and 1/2. This test is based on number of changes of sign in the series of deviations (observed value minus estimated value). The signs of deviations of discharge measurements arranged in ascending order of stage are marked for example, as shown below :

+ - + + + - - + + ...

1 1 0 0 1 0 1 0 ....

Starting from the second number in the series, mark '0' if the sign agrees or '1' if it does not agree with the sign immediately preceding. If there are N deviations in the original series, there will be (N-1) numbers of the derived series 11001010.....If the observed values could be regarded as arising from random fluctuations from the estimated values from the curve, the probability of a change in the sign could be taken as one-half. It should be noted that this assumes that the estimated value is a median rather than mean. If N is fairly large (say 25 or more), a practical criterion may be obtained by assuming that the successive signs to be independent, (that is assuming as arising only from random fluctuations) so that number of 1's or 0's in the derived sequence of (N-1) members may be judged as a binomial variable with parameters (N-1) and 1/2.

Test 2 - Test for Systematic trend in deviations

S.No.	Particulars	Symbol	Rising	Falling
1.	Number of Observations	N		
2.	Number of changes in sign	n		
3.	Probability of change	p	1/2	1/2
4.	Probability of no	q	1/2	1/2
5.	Expected number of	(N-1)p		
6.	| n - (N-P)p | - 0.5 * --------------------	t		

If the value of 't' obtained is less than 1.96 for both the curves fit for rising and falling stages, then the test confirms that there is no systematic trend in the deviations.

2.3 Test - 3

The third test is designed to find out whether a particular stage-discharge curve, on an average, yields significant under-estimates or over-estimates as compared to the actual observations on which it is based. The percentage differences i.e.,

$$\frac{(Q_O - Q_E) \times 100}{Q_E} = p$$

are worked out and averaged. If there are N observations and if p1, p2.....pi....pn are the percentage differences, and if p is the average of pi's, the standard error SE of p is given by

$$S_E = \sqrt{\frac{\sum (p-\bar{p})2}{N\,(N-1)}}$$

The average percentage p is tested against its standard error to see if it is significantly different from zero.

The percentage differences have been taken as they are rather independent of the discharge volume and are normally distributed about a zero mean value for an unbiased curve. It is to note that the tests are to be carried out for rising and falling stages separately, if different curves are used to define the stage-discharge relationships. If, however, only a single curve is used for the purpose, then the tests are to be carried out for single curve assuming both the rising and falling stage observations to form homogeneous data, as illustrated in example.

2.4 Minimum number of observations

All the above tests shall be applied to portions of curves, each individual portion being tested for bias separately. Once the bias free curve is established, it may be checked if the number of observations chosen for establishing the curve are sufficient in number. Though this test need not be applied rigorously, it can be used to have an approximate idea of the minimum number of observations required for a good stage-discharge relation within the desired degree of confidence and the reliability of the estimate desired.

The discharge observations for a particular stage are likely to show wide variation due to random errors of measurements and various other factors. It is not unusual for individual points to vary by 20% or more from the mean stage-discharge relationship. Evidently, the greater the width of the scatter band, the greater should be the number of observations necessary to ensure that the mean relationship is determined with an acceptable degree of accuracy. The variation of the percentage differences of the observed points from the curve of their mean relationship is measured by the Standard Deviation SD. The Standard Deviation is the root mean square of the percentage differences. The reliability of the mean relationship is measured by the Standard Error of the mean relationship SE which is given by SD.

The probability is approximately 20 to 1 that the shift of the apparent mean relationship (as determined from the observations) from the true relationship does not exceed 2SE. If the acceptable shift at a confidence level of 20 to 1 is set at p%, then 2SE shall not exceed p.

But SE = SD/ √N ,therefore, 2SD/ √N shall not exceed p, from which it follows that N should not be less than

{2SD/p}2

The Standard deviation shall be calculated separately for each range of stagehaving separate control. For each of these ranges, the N test should be applied separately to get the number of observations necessary to obtain a specified precision.

3.0 Conclusions

As evident from the discussions above, discharge records are developed from converting measured water stages to discharges by using a calibrated stage-discharge rating, which permits a fast and relatively inexpensive means to determine the discharge. Attempt has been made to deliberate on these complex issues to understand G&D relationship, shifting control, interpretation, tidal rating curve setting, extrapolation, and other complexities involved in G&D curve literature.

4.0 References

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